$$H(z) = H_a(s)\Big|_{s = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}}$$

EX: Transform

$$H_a(s) = \frac{s+1}{s^2+5s+6}$$

Into a digital filter using the bilinear transformation method? Choose $T_s = 1$

SOL:

$$H(z) = \frac{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 1}{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 5\cdot 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 6}$$

$$= \frac{3 + 2z^{-1} - z^{-2}}{20 + 4z^{-1} + 20z^{-2}}$$

$$\therefore\ H(z) = \frac{\overset{b_0}{0.15} + \overset{b_1}{0.1}\,z^{-1} - \overset{b_2}{0.05}\,z^{-2}}{1 + \underset{a_1}{0.2}\,z^{-1} + \underset{a_2}{z^{-2}}}$$

$$y[n] + 0.2\,y[n-1] + y[n-2]$$

$$= 0.15\,x[n] + 0.1\,x[n-1] - 0.05\,x[n-2]$$

EX: Design a digital filter with the following specs using BTM.

$$0.89125 \le |H(e^{j\omega})| \le 1 \ ; \ 0 \le \omega \le 0.2\pi$$
$$|H(e^{j\omega})| \le 0.178 \ ; \ 0.3\pi \le \omega \le \pi$$

SOL:

$$\Omega = \frac{2}{T_s} \tan\left(\frac{\omega}{2}\right)$$

$T_s = 1$; for convience.

$\therefore$ the corrisponding analog filter specs will be

$$0.89 \le |H_a(j\omega)| \le 1$$

$$0 \le \Omega \le 2\tan\left(\frac{0.2\pi}{2}\right)$$

$$|H_a(j\Omega)| \le 0.178$$

$$2\tan\left(\frac{0.15\pi}{2}\right) \le \Omega \le \infty$$

Because of mentonic respones,

$$H_a(j2\tan 0.1\pi) = 0.89$$

$$H_a(j2\tan 0.15\pi) = 0.178$$

We know

$$|H_a(j\Omega)|^2 = \frac{1}{1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

$$\therefore \quad 1 + \left(\frac{2\tan(0.1\pi)}{\Omega_c}\right)^{2N} = \frac{1}{0.89^2} \qquad (1)$$

$$1 + \left(\frac{2\tan(0.15\pi)}{\Omega_c}\right)^{2N} = \frac{1}{0.178^2} \qquad (2)$$

Solving (1) & (2)

$$N = \frac{\log\left[\dfrac{\frac{1}{0.178^2} - 1}{\frac{1}{0.89^2} - 1}\right]}{2\log\left[\dfrac{\tan 0.15\pi}{\tan 0.1\pi}\right]}$$

$$= 5.305 \cong 6$$

must be next highest intergar.

Putting $N=6$ into equ (2) we get $\Omega_c = 0.766$ For this value of Ω_c the pass band specs are met exactly. This is reasobrable with the BTM. as we don't have to be concerned with aliasing.

Similiar to example 7.2 to get $H_a(s)$

$$H_a(s) = \frac{0.2024}{(s^2 + 0.3995 + 0.5871)(s^2 + 1.0836s + 0.5871)(s^2 + 1.4802s + 0.5871)}$$

$$H(z) = H_a(s)\Big|_{s = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}}$$

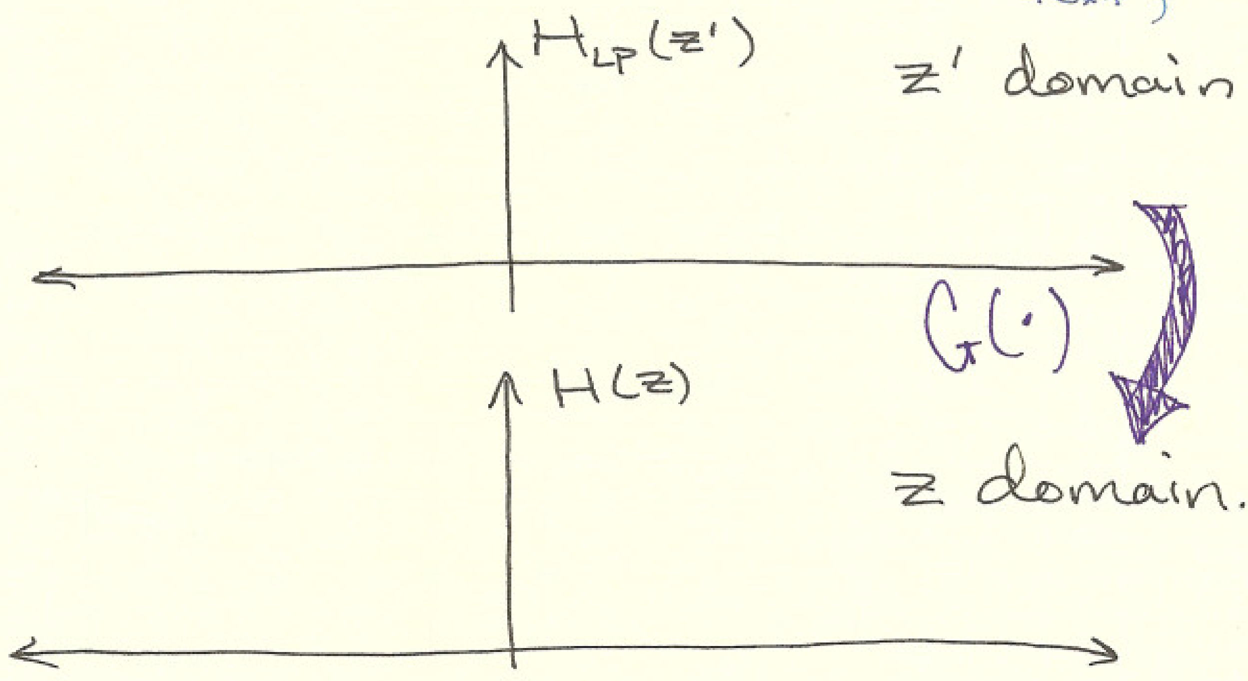
* use Butterworth polynomial table to get $H_a(s)$.

To create a different type filter, we create a LP prototype, then we go to the transformation table

4.3 Frequency Band Transformation

Design of H.P. B.P. and B.S. filters from a prototype lowpass filter

(CH 10 from text)



$z' = e^{j\omega'}$ $\quad\quad z = e^{j\omega}$

let $H_{LP}(z')$ is TF of the prototype filter
$H(z)$ is " " " desired "

Assume that $H_{LP}(z')$ is a causal and stable filter. We want $H(z)$ to be stable and causal. This requires the following conditions

1. The unit circle in z' plane must map onto the unit circle of the z plane.

2. The inside unit circle in z' plane must map onto the inside of the circle in z-plane

According to 1

$$|z^{-1}| = |G(z^{-1})| = |G(e^{-j\omega})| = 1$$

$$e^{-j\omega'} = G(e^{-j\omega}) = |G(e^{-j\omega})| \angle G(e^{-j\omega})$$

$$= |G(e^{-j\omega})| e^{j\angle G(e^{-j\omega})}$$

$$\therefore \quad -\omega' = \angle G(e^{-j\omega})$$

Based on these relations the transformation. The general form of the function $G(\cdot)$ that satisfy all of the above requirements is a rational function of all pass types given by

$$z'^{-1} = G(z^{-1}) = \pm \prod_{k=1}^{n} \left(\frac{z^{-1} - \alpha_k}{1 - \alpha_k z^{-1}} \right)$$

Where, $|\alpha_k| < 1$ for stability req. 2
n and α_k will vary depending on the

type of desired filter

Handout → Table 8.2

Text → Table 10.2

☆ We should know LP to LP and LP to HP. ☆

EX: Consider the system function of a prototype LP filter is given as $H_{LP}(z')$

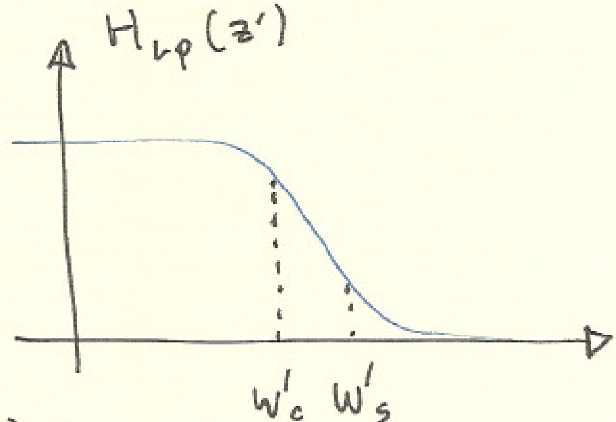
$$H_{LP}(z') = \frac{0.001836\,(1+z'^{-1})^4}{(1-1.5z'^{-1}+0.848z'^{-2})(1-0.155z'^{-1}+0.6493z'^{-2})}$$

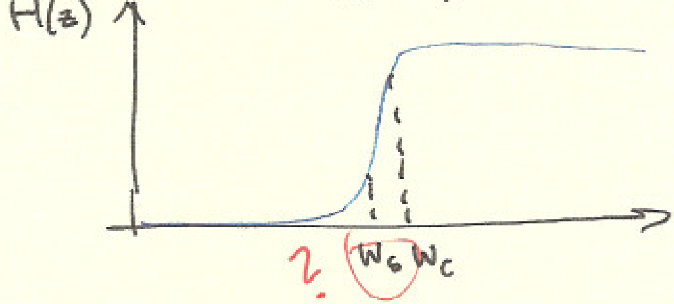
and $\omega'_c = 0.2\pi$

$\omega'_s = 0.3\pi$

Design a HP filter with the same tolerance but cut off freq at $\omega_c = 0.6\pi$

SOL:





Using table 8.2 ....

$$\alpha = -\frac{\cos\left(\frac{\omega'_c + \omega_c}{2}\right)}{\cos\left(\frac{\omega'_c - \omega_c}{2}\right)}$$

$$= -0.382$$

$$H(z) = H_{LP}(z')\Big|_{z'^{-1} = \frac{z^{-1} - 0.382}{1 - 0.382 z^{-1}}}$$

$$H(z) =$$